Lecture 5. **Typical dynamic links**

A link itself stays for a mathematical model of an element, combination of elements or any other subpart of a system. A dynamical system can be presented as a combination of typical (or basic) elements having differential equations describing them of order not more than two.

1) *The Integrating Link*

It is described by the following equation:

$$T\frac{d\theta_{out}}{dt} = K\theta_{in}(t) \quad \text{or} \quad T\dot{\theta}_{out} = K\theta_{in}(t) \;. \tag{2.27}$$

The transfer function of the link is $W(s) = \frac{K}{Ts}$, where $K \geq 0; T > 0$.

The characteristic equation is $Q_2(s) = Ts = 0$, $T \neq 0$; hence, it has unique solution $s \equiv 0$.

The basic property is to remember the signal for an infinitely long period of time. An example of the link: an integrator without feedback.

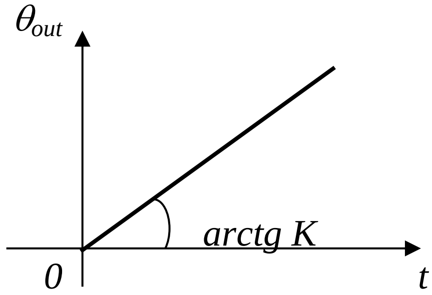

Fig. 2.3. An output coordinate of the integrating link

In state space the link is described by the system: $\begin{cases} \dot{x} = bu \\ y = cx \end{cases}$,

where $b = K/T$, $c = 1$.

2) *The first order Aperiodic Link (Inertial Link)*

It is described by the following equation:

$$T\dot{\theta}_{out} + \theta_{out}(t) = K\theta_{in}(t) \tag{2.28}$$

where $K > 0$; $T > 0$.

The transfer function is $W(s) = \frac{1}{Ts+1}$.

The characteristic equation is $Q_2(s) = Ts + 1 = 0$; hence $s = -\frac{1}{T}$. From here we can solve for θ_{out}(t): $\theta_{out}(t) = c_1 e^{-t/T}$, were c_1 is the constant of integration.
The transition time for the link is equal to $(3 \div 4)T$, where T is the integration constant. A feedback integrator is an example.

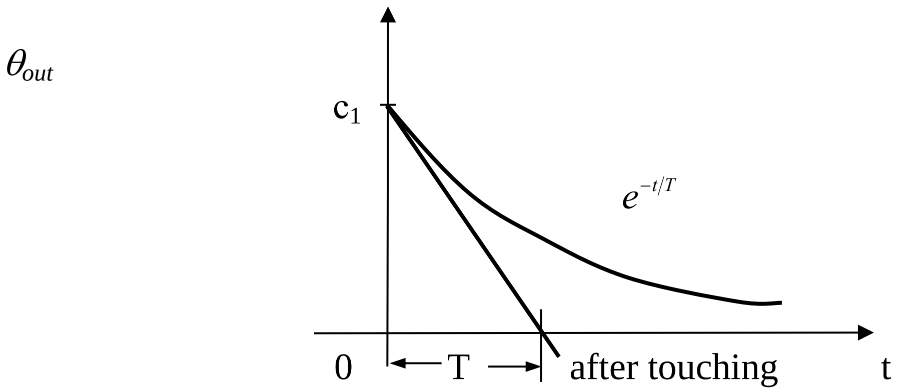

Fig. 2.4. An output coordinate of the first order aperiodic link

In state space the link is described by the system:

$$\begin{cases} \dot{x} = Ax + bu \\ y = Cx \end{cases}, \quad \text{where} \quad A = \left| -\frac{1}{T} \right|; \; b = \frac{K}{T}; \; C = 1.$$

3) *The Oscillating Link* (damped oscillations)

It is described by the following equation:

$$T^2 \ddot{\theta}_{out} + 2\xi T \dot{\theta}_{out} + \theta_{out}(t) = K\theta_{in}(t); \; 0 < \xi < 1 . \qquad (2.29)$$

The transfer function is $W(s) = \frac{K}{T^2 s^2 + 2\xi Ts + 1}$.

The characteristic equation is $Q_2(s) = T^2 s^2 + 2\xi Ts + 1 = 0$.

The corresponding complex conjugate roots are

$$S_{1,2} = -\alpha \pm j\beta; \; S_{1,2} = -\frac{\xi}{T} \pm \frac{\sqrt{\xi^2 - 1}}{T} \; ; \; 0 < \xi < 1 .$$

Solving for θ_{out} *(t)* gives us $\theta_{вых}(t) = C_2 e^{-\alpha t} \cos(\beta t + \varphi)$ where $\varphi = arctg \frac{\beta}{\alpha}$; $C_2 = const.$
The link characteristics:

a) Damping factor: $e^{-\alpha t}$.

b) Oscillation period: $T = \dfrac{2\pi}{\beta}$.

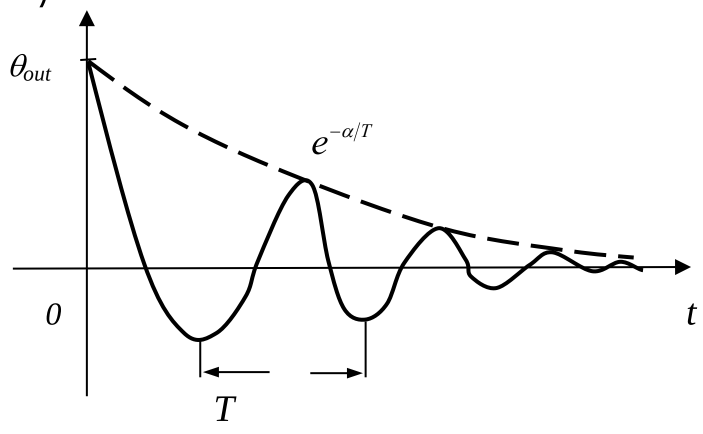

Fig. 2.5. An output coordinate of the oscillating link

In state space the link is described by the system:

$$\begin{cases} \dot{x} = Ax + bu \\ y = Cx \end{cases},$$

where $A = \begin{vmatrix} 0 & 1 \\ -\dfrac{1}{T^2} & -\dfrac{\xi}{T} \end{vmatrix}$; $B = \begin{vmatrix} 0 \\ \dfrac{K}{T^2} \end{vmatrix}$; $C = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$.

An example: RLC-circuits (active-inductance-capacitance-chain) having damped sinusoid output.

Pay attention to the fact that keeping ξ in (2.29) in the range ($0< \xi< 1$) produces the oscillating link, keeping $\xi=1$ produces the *aperiodic link of the second order* (see description later in the book). $\xi=0$ gives us the *conservative link.*

4) *The Conservative Link* (free oscillations).

It is described by the following equation:

$$T^2\ddot{\theta}_{out} + \theta_{out}(t) = K\theta_{in}(t). \qquad (2.30)$$

The transfer function is $W(s) = \dfrac{1}{T^2 s^2 + 1}$.

The characteristic equation is $Q_2(s) = T^2 s^2 + 1 = 0$ where imaginary roots: $s_{1,2} = \pm j\beta$; $s_{1,2} = \pm j\dfrac{1}{T}$.

Solving for $\theta_{out}(t)$ gives us $\theta_{out}(t) = c_3 \cos \omega t$ where $c_3 = const.$

In state space: $\begin{cases} \dot{x} = Ax + bu \\ y = Cx \end{cases}$,

where $A = \begin{vmatrix} 0 & 1 \\ -\dfrac{1}{T^2} & 0 \end{vmatrix}$; $B = \begin{vmatrix} 0 \\ \dfrac{K}{T^2} \end{vmatrix}$; $C = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$.

$$h(t) = K\,(1 - \cos \omega_1 t)\ ,\quad \omega_1 = 1/T. \qquad (11)$$

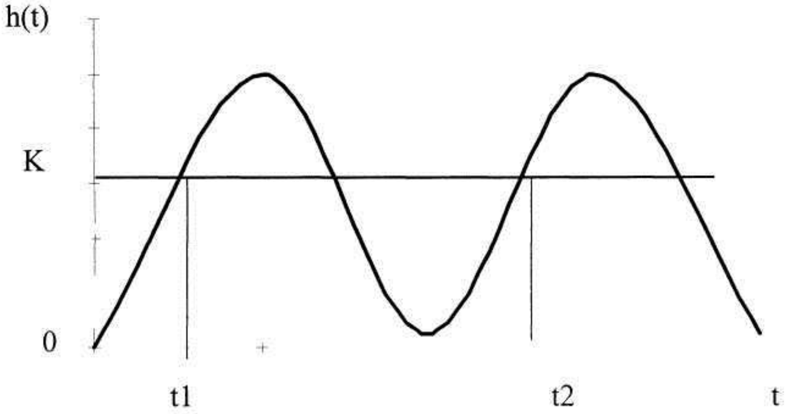

Fig. 2.6. An output coordinate of the c*onservative* link

An example: RLC-circuits having stable sinusoidal output with constant amplitude.

The typical dynamic links presented above are of large importance in control theory, since they provide the way of describing dynamic systems of arbitrary complexity in terms of comparatively simple and well-studied building blocks.

Having described internal structure of dynamic systems, we now will turn our attention to their external part, namely, typical input actions, distinguished in control theory. In general, they are classified into two categories: determinate and accidental (casual).